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Signal Processing for Micro Inertial Sensors

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Abstract: In the development of the guidance and control packages for unmanned vehicles, it is highly desirable to have inertial measurement sensors which are small, inexpensive, low power, reliable and accurate. New technological advances in the design and construction of micro inertial sensors, such as accelerometers and gyroscopes, have much promise in providing small, inexpensive, and low power devices; however, much improvement in the reliability and, especially, the accuracy of these micro devices is still necessary. Further major improvements in these two properties will probably not be accomplished in the near future, thus it will be necessary to use special signal processing methods to provide the accuracy. One way which has been proposed to improve the accuracy, and concurrently the reliability, of micro sensors is to use many, perhaps one hundred or more, micro sensors on a single chip (or a few chips) and using statistical methods to combine the individual outputs of these sensors to provide an accurate measurement. One method of performing such a combination is through an extended Kalman filter (EKF). A standard application of an EKF to an array of gyroscopes would involve at least six state equations per gyroscope and the number of covariance equations would be in the order of the square of the product of six times the number of gyroscopes. Obviously, the 'curse of dimensionality' very quickly limits the number of sensors (gyroscopes) which can be used. Even if the EKF for each individual gyroscope is uncoupled from the rest, the number of covariance equations is of the order of the number of gyroscopes times six squared. This can still lead to a formidable computational burden. In this paper, a new technique of applying an EKF to this problem of combining many sensors is proposed. By using the common nominal model for each of the micro sensors and developing a single EKF, improved accuracy is achieved by a single EKF with the dimension of one sensor. For cases in which the micro sensors are corrupted by correlated noise (between the sensors) an artificial neural network could be added to the EKF dynamics to track the noise. Simulated examples will be discussed.

Summary: Recent advances in the technology of microelectromechanical systems (MEMS) has led to optimistic predictions of the potential applications of various micro devices. One family of devices which appears to have wide applicability in many areas is the family of inertial sensors, including gyroscopes and accelerometers. Micro-sensors, because of their small size, often have only moderate accuracy as compared to the accuracy of full-size sensors. Applications which demand sensors with small size and low cost as well as high accuracy, will require signal processing methods. Because of the expected low cost of the micro-sensors, one approach to increased accuracy is to use a large number of micro-sensors to measure the same quantity and then use statistical methods to combine the many low accuracy measurements to generate a single high accuracy measurement. For many years the extended Kalman filter (EKF) has been used to process output signals from inertial sensors to produce more accuracy than is possible from the raw output signals, thus it is natural to assume that the EKF can be readily adapted to the many micro-sensor problem. In this paper, a technique is proposed in which an arithmetic average of the many outputs from a set of micro-gyroscopes is input to a single average EKF. Under certain simple assumptions, this can be shown to produce the desired result. Using a model of a real micro-gyroscope this technique was simulated for a set of ten micro-gyroscopes and the preliminary results indicate that this method has promise. Continuing exploration of this method will include adaptive EKF methods using a neural network based EKF which will allow the filter to track correlated errors.

Introduction. Recent developments in micro-electromechanical systems (MEMS) technology have led to optimistic predictions for the use of micro devices in a wide range of applications. It appears that micro-sensors, in particular, have immediate applicability in many fields from medical implants to automobiles to aerospace vehicles. Small size plus their probability of becoming very inexpensive make their potential range of applications almost limitless. However, in spite of small size being a major advantage of micro-sensors, it can also be a major disadvantage. In particular, in many cases, the accuracy of a sensor can be controlled much easier if the sensor has large dimensions, because accuracy is often a function of dimensional ratios and controlling such ratios by machining is easier when dimensions are larger. On the other hand, if the micro-sensors can be made much more inexpensively than larger machined sensors then this disadvantage may be overcome by using many relatively inaccurate, but also inexpensive, sensors as opposed to one, or a few, accurate, but much more expensive, sensors. Note that the use of many micro-

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sensors measuring the same quantity can also provide reliability through redundancy at a reasonable cost. This latter point, while of considerable interest, will not be considered in this paper. The problem of interest here is that of combining the outputs of several micro-sensors, all measuring the same quantity, so that the accuracy of the combination exceeds the accuracy of the individual micro-sensors but without requiring an extraordinary amount of computation. This is what will be referred to as the signal processing problem for micro inertial sensors.

The signal processing problem: A sensor is a device that measures a physical quantity and produces a corresponding output, typically an electrical quantity, which is related in a known way to the physical quantity. In practice, the measurement of the physical quantity is corrupted by noise and the relationship between the physical quantity and the corresponding output is corrupted by another noise. The accuracy of the sensor is dependent upon the magnitudes of these noises. The accuracy of a sensor might be improved by better understanding of the physics by which the sensor measures the physical quantity and how it produces the corresponding output and improving the process by which the sensor is manufactured or by applying appropriate signal processing techniques to the output signal. In the area of micro-sensors, significant accuracy improvements in the manufacturing process will require increased costs and/or larger geometries, both of which will tend to neutralize the advantages of such devices. Thus signal processing techniques applied to multiple sensor configurations are being examined and are expected to improve the accuracy of existing micro-sensors. This is not to imply that improved manufacturing processes are not being developed; however, currently, appropriate processing of micro-sensor outputs appears to be the quickest and most feasible method of improving sensor accuracy.

The simplest signal processing concept to improve micro-sensor accuracy is to manufacture one (or a few) chips with a total of many micro-sensors all of which measure the same physical quantity. The outputs of all of the micro-sensors are then simply averaged arithmetically. If the noises associated with the outputs of the several micro-sensors are additive, mutually independent and zero mean, then the standard deviation of the error of the arithmetic average is reduced by the square root of the number of output signals. Thus, if one hundred micro-sensors are used, it is equivalent to replacing them with a single micro-sensor with an error with one-tenth the standard deviation. However, if the errors in the output signals are non-zero mean (across the ensemble of micro-sensors) or if the errors are correlated across the micro-sensors, then arithmetic averaging may not be effective. That is, the averaging will reduce the uncorrelated component of the noise but may have little effect on the correlated component of the noise. A correlated component of the noise in the output signals is to be expected if many of the sensors are on a single chip. For example, any alignment errors in the chip manufacturing process may generate measurement errors in one micro-sensor which are strongly correlated with all of the measurement errors on the other micro-sensors. If the manufacturing process generates alignment errors which are correlated from chip to chip, for example all the chips come from a single wafer, then this correlation may extend to all of the micro-sensors. In this case, simple arithmetic averaging may not be effective as a signal processing technique.

In the application of a standard sized inertial sensor, it is quite common to process the output of the sensor through a Kalman filter, or more likely through an extended Kalman filter (EKF), to improve the accuracy of the measurement. If a number of inertial sensors, for example, three gyroscopes and three accelerometers, are combined on a single platform it is common that a single extended Kalman filter is used to generate estimates of the state of the platform on which the sensors are mounted. These estimates are generally much improved over the measurements which are taken directly from the sensors. A similar concept could be used to generate an estimate of the common quantity being measured by the many micro-sensors. In particular, micro inertial sensors have well developed mathematical models and are well suited to the application of an extended Kalman filter. The difficulty with applying this concept to a set of many micro inertial sensors is that the dimensionality of the EKF may become excessive. For example, if a single gyroscope is modeled by a set of six coupled dynamic equations, then six hundred equations would be needed to model one hundred gyroscopes. The resultant EKF which uses this dynamic model would have six hundred states and its covariance equation would have between two thousand and one hundred eighty thousand (180,000) equations depending upon the coupling between the various gyroscopes. Obviously, this method of combining measurements is limited by the number of micro gyroscopes which are being used.

The method proposed in this paper for combining the measurements from many micro-sensors is to first generate the arithmetic average of the outputs of the sensors and then process this average output through an average EKF. Consider the following standard Kalman filter problem of estimating the state of a system defined by the vector-matrix equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{w} \qquad (1)$$

where $\mathbf{x}$ is the system state vector, $\mathbf{u}$ is the system input vector, and $\mathbf{w}$ is a zero mean white noise vector with covariance $\mathbf{Q}$. $\mathbf{A}$ and $\mathbf{B}$ are the matrices which define the system. The state vector is measured through a noisy linear transformation

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{v} \qquad (2)$$

where $\mathbf{C}$ defines the transformation and $\mathbf{v}$ is a zero mean white noise with covariance matrix $\mathbf{R}$. Suppose that the system state can also be measured from a set of N other systems defined by

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{A}\mathbf{x}_i + \mathbf{B}\mathbf{u} + \mathbf{w}_i, \quad i = 1,2,\cdots,N$$

where $\mathbf{x}_i$ is the state vector and $\mathbf{w}_i$ is the white noise vector of the ith system. The states of these systems are measured through a set of N measurement equations

$$\mathbf{y}_i = \mathbf{C}\mathbf{x}_i + \mathbf{v}_i, \quad i = 1,2,\cdots,N$$

where $\mathbf{v}_i$ is the white noise of the ith measurement. An arithmetic average of the outputs is formed as

$$\overline{\mathbf{y}} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{y}_i = \mathbf{C}\left(\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i\right) + \frac{1}{N}\sum_{i=1}^{N} \mathbf{v}_i$$

or

$$\overline{\mathbf{y}} = \mathbf{C}\overline{\mathbf{x}} + \overline{\mathbf{v}} \qquad (3)$$

Now averaging the N differential equations results in the average differential equation

$$\frac{d\overline{\mathbf{x}}}{dt} = \mathbf{A}\overline{\mathbf{x}} + \mathbf{B}\mathbf{u} + \overline{\mathbf{w}} \qquad (4)$$

In the stochastic sense equation (3) is identical to equation (2) and equation (4) is identical to equation (1), except that the noise $\overline{\mathbf{w}}$ has a covariance of $\mathbf{Q}/N$ and the noise $\overline{\mathbf{v}}$ has a covariance of $\mathbf{R}/N$. Thus if estimating the system response to the input $\mathbf{u}$ is of primary interest, the effect of the white noises is reduced if the average system output is used to drive the Kalman filter rather than the measurement in equation (2). In the following example, the problem of estimating the output of a set of ten micro-gyroscopes all measuring the same angular rate using a single EKF was simulated and compared with the method of applying ten EKFs with output averaging

Mathematical model of a micro-gyroscope: In this section, the mathematical model of a micro gyroscope is presented. The parameters used later in the simulation are taken from an actual experimental micro-gyroscope developed by Irvine Sensors, thus the simulation represents the real world as nearly as we were able to model it. The gyroscope is composed of a small rigid plate attached to a housing through a set of four thin orthogonal support wires. A set of orthogonal axes is fixed to the plate with the origin at the center of mass of the plate and the three axes are aligned with the principal axes of the plate. The x-axis and the y-axis lie in the plane of the plate and are assumed to be aligned with the support wires. In an application, the housing is attached firmly to a much larger rigid body (perhaps an automobile), thus the plate can rotate, to a limited extent, about all three axes relative to the housing and, thus also rotates relative to the larger rigid body. However, it is assumed that that any translation relative to the larger rigid body is negligible. Because the axes rotate with the plate, the products of inertia of the plate in this coordinate system are zero. The plate is forced to oscillate at a constant amplitude and frequency about the z-axis by a sinusoidal torque applied by an electric field. The x-axis is the input axis through which the input angular velocity of the large rigid body is coupled into the plate through the support wires. The y-axis is the output axis of the gyroscope from which a measurement of the input angular velocity is obtained as some function of the periodic motion induced about the y-axis by the angular motions about the x-axis and the z-axis. Under these conditions, Euler's equations of motion can be used to develop a dynamic model of the plate. The plate is symmetric in the x-y plane so that the moments of inertia about the x-axis and the y-axis are equal. The moment of inertia about the z-axis is much larger than these two. We define the angular position of the plate from its 'rest' position by the angles θ_x, θ_y, and θ_z measured about the x, y, and z axes. A sinusoidal torque, $T(t)$, is applied to the plate about the z-axis producing a periodic motion about the z-axis. The other torques about the z--axis are a damping torque and a spring torque, both due to the mechanical properties of the supporting wires along the x-axis and the y-axis. The applied torques about the y-axis and the x-axis are a damping torque and a spring torque due to the supporting wires and the damping coefficients and spring constants are assumed to be equal about each axis because of the symmetry of the plate in the $x-y$ plane. When the housing is rotated at an angular rate of Ω_x rad/sec about the x-axis, a torque $T_x(t)$ is transmitted to the plate by the support wires and consists of a damping torque and a spring torque

$$C_x(\Omega_x - \dot{\theta}_x), \quad \text{and} \quad K_x\left(\int_0^t \Omega(t')dt' - \theta_x\right)$$

where C_x and K_x are the damping coefficient and the spring constant about the x-axis. The input rate Ω_x induces a motion in the y-axis and Ω_x is determined from the measurement of the angular motion about the y-axis. Based on these conditions , Euler's equations can be written as

$$\ddot{\theta}_x + a_1\dot{\theta}_x + a_0\theta_x + A\dot{\theta}_y\dot{\theta}_z = a_1\Omega_x + a_0\int_0^t \Omega_x(t')dt'$$

$$\ddot{\theta}_y + a_1\dot{\theta}_y + a_0\theta_y - A\dot{\theta}_x\dot{\theta}_z = 0$$

$$\ddot{\theta}_z + b_1\dot{\theta}_z + b_0\theta_z = T_0 \sin\omega_o t$$

where the coefficients are determined from the various physical parameters of the gyroscope. The desired output of the gyroscope is the input angular rate output $\Omega_x(t)$. This is obtained by measuring and processing the angular rate $\dot{\theta}_y(t)$.

Steady-State Operation: Assuming that the rate of change of the angular rate input $\Omega_x(t)$ to the gyroscope is slow relative to the rates of change induced by the applied torque $T_0 \sin\omega_0 t$ and that the nonlinear effects in the x and y equations are relatively small, phasor analysis can be used to determine a steady-state value of the output $\dot{\theta}_y(t)$ as

$$\dot{\theta}_y(t) = AT_0 G_z(\omega_0) G_y(\omega_0)\omega_0^2 \Omega_x \sin\omega_0 t$$

where

$$G_z(\omega_0) = \frac{1}{\sqrt{(b_0 - \omega_0^2)^2 + (\omega_0 b_1)^2}}$$

and

$$G_y(\omega_0) = \frac{1}{\sqrt{(a_0 - \omega_0^2)^2 + (\omega_0 a_1)^2}}$$

Note that the phase shift of the phasor is not needed and has been ignored. Note that this equation could be written

$$\dot{\theta}_y(t) = K\Omega_x(t)\sin\omega_0 t$$

where K is a constant. Thus $\dot{\theta}_y(t)$ is sine wave $\sin\omega_0 t$ modulated by the applied angular rate $\Omega_x(t)$. The angular rate can thus be obtained by a simple amplitude modulation (AM) demodulator.

State Equations for the Gyroscope: For a Kalman filter the equations for the gyroscope must be written in the form of state equations. To put the three equations into a state equation form, let

$$x_2 = \dot{\theta}_x$$

$$x_3 = \theta_y \qquad x_4 = \dot{x}_3 = \dot{\theta}_y$$

$$x_5 = \theta_z \qquad x_6 = \dot{x}_5 = \dot{\theta}_z$$

then the state equation representation for the gyroscope is

$$\dot{x}_1 = -a_0 x_2 + a_0 \Omega_x$$

$$\dot{x}_2 = x_1 - a_1 x_2 - A x_4 x_6 + a_1 \Omega_x$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -a_0 x_3 - a_1 x_4 + A x_2 x_6$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = -b_0 x_5 - b_1 x_6 + T_0 \sin\omega_0 t$$

where the variable x_1 has no specific physical meaning. Two additional equations can provide a simple demodulation of the variable

$$x_4(t) = \dot{\theta}_y(t)$$

to generate $\Omega_x(t)$. These equations are

$$\dot{x}_7 = x_8$$

$$\dot{x}_8 = -c_0 x_7 - c_1 x_8 + c_0 x_4 \sin\omega_0 t$$

where the coefficients were chosen to provide low pass filtering with a corner frequency at $\omega_0 / 10$ rad/sec. The output measurement is now the modulator output, that is,

$$z(t) = x_7(t)$$

Ideally, in steady state, this is related to $\Omega_x(t)$ by the relationship developed in the steady-state section and the demodulator gain by

$$\Omega_x(t) = \frac{K}{2} x_7(t)$$

Design Coefficients. The design coefficients for the gyroscope simulated in this paper are

$$a_1 = 1.8621 \times 10^4$$

$$a_0 = 3.4483 \times 10^8$$

$$b_1 = 9.3750 \times 10$$

$$b_0 = 3.3750 \times 10^8$$

$$A = 4.5172$$

and

$$\omega_0 = 2\pi \times 3000 = 1.8850 \times 10^4 \text{ rad / sec}$$

which is near the resonant frequency of the linear part of the y-axis equation. The demodulator coefficients are

$$c_1 = 1200\pi \text{ and } c_0 = (600\pi)^2$$

The magnitude T_0 of the forcing function was chosen to be

$$T_0 = 1.5 \times 10^8$$

which generates a maximum steady state motion about the x-axis of a fraction of a radian The resulting state equations using these coefficients was used as the 'truth model' for the simulations which are discussed later.

Kalman Filter Equations. To generate a set of Kalman filter equations, the following modification to the gyroscope state equations had to be made. Let:

$$\Omega_x = x_2 + n_1$$

where n_1 is the unknown difference between the input Ω_x and the state $x_2 = \dot{\theta}_x$. The resulting state equations can now be rewritten:

$$\dot{x}_1 = a_0 n_1$$
$$\dot{x}_2 = x_1 - Ax_4x_6 + a_1 n_1$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -a_0x_3 - a_1x_4 + Ax_2x_6 + n_4$$
$$\dot{x}_5 = x_6$$
$$\dot{x}_6 = -b_0x_5 - b_1x_6 + T_0 \sin\omega_o t + n_6$$

with the demodulator equations

$$\dot{x}_7 = x_8$$
$$\dot{x}_8 = -c_0x_7 - c_1x_8 + c_0x_4 \sin\omega_0 t + n_8$$

The measurement equation is given by

$$z = x_7 + v$$

The quantities $n_1, n_4, n_6, \mathrm{n}_8$ and v are random quantities which in the development of the extended Kalman filter (EKF) are assumed to be white noises. As almost four decades of usage has shown the EKF is robust to inaccuracies in the assumptions on the system noises. For convenience, the gyroscope model is now rewritten in the vector-matrix form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}\mathbf{n} + \mathbf{g}(\mathbf{x})\sin\omega_0 t$$
$$z = \mathbf{h}^T\mathbf{x} + v$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \quad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_0 \\ 0 \\ c_0x_4 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} a_0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{n} = \begin{bmatrix} n_1 \\ n_4 \\ n_6 \\ n_8 \end{bmatrix}$$

and

$$\mathbf{f}(\mathbf{x}) = \mathbf{F}\mathbf{x} + \mathbf{f}_1(\mathbf{x})$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_0 & -a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b_0 & -b_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_0 & -c_1 \end{bmatrix}$$

and

$$\mathbf{f}_1(\mathbf{x}) = \begin{bmatrix} 0 \\ -Ax_4x_6 \\ 0 \\ Ax_2x_6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The noise statistics, which are normally determined empirically, are chosen to have the following properties, R is a nonzero, positive scalar and $\mathbf{Q}$ is a nonzero, positive definite 4×4 matrix assumed to be diagonal and given by:

$$\mathbf{Q} = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix}$$

where $q_{11} > 0$ $q_{22} > 0$ $q_{33} > 0$ $q_{44} > 0$

The extended Kalman filter equations can now be written as

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{k}(z - \mathbf{h}^T\hat{\mathbf{x}}) + \mathbf{g}(\hat{\mathbf{x}}) \cdot \sin\omega_0 t$$

where $\hat{\mathbf{x}}$, the output of the Kalman filter, is the estimate of the combined state of the gyroscope and the demodulator. The estimate of the applied torque $\Omega_x(t)$ is given by the state estimate $\hat{x}_7(t)$. $\mathbf{k} = \mathbf{P}\mathbf{h} / R$ is the Kalman gain vector and the error covariance $\mathbf{P}$ is calculated by the equation

$$\dot{\mathbf{P}} = \overline{\mathbf{F}}\mathbf{P} + \mathbf{P}\overline{\mathbf{F}} - \frac{\mathbf{P}\mathbf{h}^T\mathbf{P}}{R} + \mathbf{G}\mathbf{Q}\mathbf{G}^T$$

where

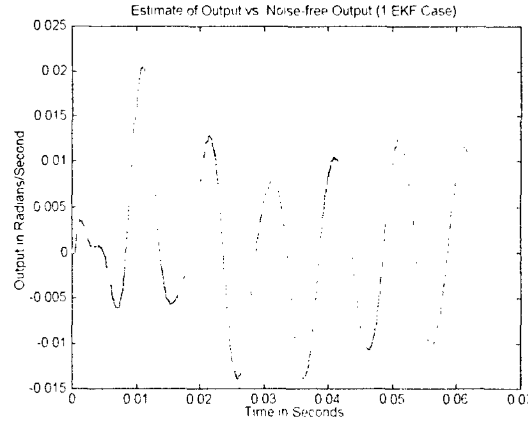
$$\overline{\mathbf{F}} = \mathbf{F} + \mathbf{F}_1$$

and

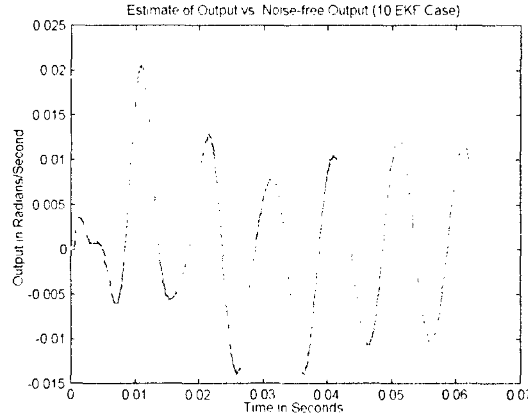
$$\mathbf{F}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A\hat{x}_6 & 0 & -A\hat{x}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A\hat{x}_6 & 0 & 0 & 0 & A\hat{x}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These equations were used to simulate the EKFs used in the simulations.

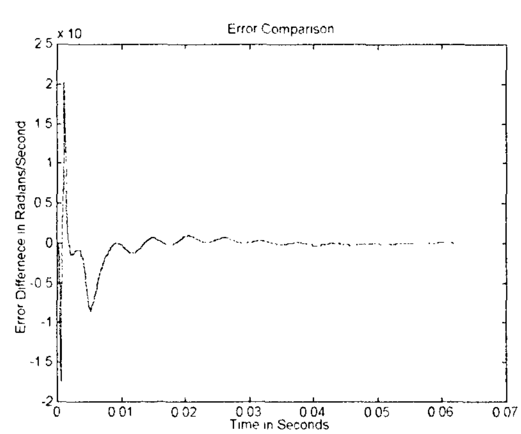
Simulation results: The technique described in this paper of averaging the output measurements from several micro-gyroscopes and processing the average output in a single EKF to estimate the common angular rate applied to each micro-gyroscope was simulated using ten micro-gyroscopes with the micro-gyroscope model described earlier. The applied angular rate input $\Omega_x(t)$ was a sine wave with a maximum amplitude of one-tenth rad/sec and a frequency of one hundred Hertz .A measurement noise with a standard deviation of twenty per cent of the maximum of the steady-state value (.012) of the output signal of the truth model driven by this input was used to corrupt the output of each micro-gyroscope. The output of this simulation was compared to the simulated output of the truth model. The results are shown in the figure below in which the solid line is the noise free output and the dotted line is the estimate. The standard deviation of the time average of the steady state error is $(2.3672)10^{-4}$. This is an improvement of more than ten times over the given measurement noise. With simple averaging, an improvement of $\sqrt{10}$ would be expected.



The technique of developing individual estimates of the applied angular rate using one EKF for each of the ten micro-gyroscopes and averaging the outputs was also simulated and compared to the truth model. These results are shown in the figure below in which the dotted line is the noise-free output and the solid line is the estimate. The standard deviation of the time average of the steady state error is $(2.1058)10^{-4}$.



Finally, the two multiple sensor techniques were compared to each other. The error, compared to the truth model, was generated for each and the difference between these two errors is shown in the figure below.



It is apparent that the two different signal processing methods are very comparable. There seems to be only a slight degradation of the estimate when one EKF is used as opposed to ten EKFs. On the other hand, the amount of computation is considerably reduced.

Conclusions: In this paper a method of improving the accuracy of micro inertial sensors is proposed. This technique uses multiple sensors with advanced signal processing techniques to combine the many signals into a single output. Two such techniques are discussed, the first is very computationally intensive and the second is moderately computationally intensive. In order to be useful, the outputs generated by these techniques should be significantly more accurate than the output of a single sensor or even of the arithmetic average of the outputs of many sensors. Some preliminary simulation results indicate that both techniques improve the sensor accuracy significantly; however, the computational intensity of the first technique increases exponentially with the number of sensors, which precludes it from being used with a very large number of sensors. In the second technique, the computational burden is independent of the number of sensors and if it has comparable accuracy to the first technique then is far preferable. Early comparisons of the two techniques indicates that the accuracies are comparable.

This research is still in a preliminary stage and definitive conclusions are premature; however the early simulations give results that are promising enough to encourage further work in this area.